Department of Pure and Applied Mathematics

PART A

Research Methodology

General and Scientific Aptitude: General Information of Science and Its interface with society to test the candidate's awareness of science aptitude of scientific and quantitative reasoning.

Fundamentals of Computer: Generations of Computer (I-V), Functions of the Different Units: Input unit Output unit, Memory unit, CPU (ALU+CU), Input & Output Devices Memories: Registers, Cache, Primary Memory, Secondary Memory Software: System Software, Application Software, MS Office: Understanding Word Processing, Using Spread Sheet, Making Presentations Introduction to Internet, WWW and Web Browsers, introduction to networks, E-mail, Number System: Decimal, Octal, Hexadecimal, Base conversions, Binary Codes: BCD.

Statistics and Probability: Frequency distribution, Measures of central tendency Probability: event, sample space, mutually exclusive, independent event, Permutation and Combinations, addition theorem, multiplication theorem, conditional probability, Baye's theorem, Random variables, Probability Distribution: Binomial, Poisson and Normal Distributions, Multivariate Analysis and Curve Fitting: Correlation, Correlation Coefficient, Rank Correlation, Linear Regression, Principles of Least Squares and Curve Fitting.

Matrices: Definition and examples, different type of matrices, Inverse of matrix, Rank of Matrix, Eigen values and Eigen vectors of matrix, Characteristic Equation, Crammer's Rule and Gauss Jordon method for the solution of Simultaneous equations.

Discrete Mathematics: Sets, Operations on sets, Relations and functions, Power set, Induced maps, Number system, The Peano's axioms, Cardinality of sets, Mathematical Induction; Statement Logic: Statement and Notations, Connectives, Conjunctive normal forms, Disjunctive normal forms, Statement Calculus, Lattices, Boolean algebra.

PART B

UNIT - I

Analysis:

Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, limsup, liminf. Bolzano Weierstrass theorem, Heine Borel theorem. Continuity, uniform continuity, differentiability, mean value theorem. Sequences and series of functions, uniform convergence. Riemann sums and Riemann integral, Improper Integrals. Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral. Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation, inverse and implicit function theorems. Metric spaces, compactness, connectedness. Normed linear Spaces. Spaces of continuous functions as examples.

Complex Analysis:

Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions. Analytic functions, Cauchy-Riemann equations. Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem. Taylor series, Laurent series, calculus of residues. Conformal mappings, Mobius transformations.

UNIT - II

Linear Algebra:

Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices, linear equations. Eigenvalues and eigenvectors, Cayley-Hamilton theorem. Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic forms.

Algebra:

Permutations, combinations, pigeon-hole principle, inclusion-exclusion principle, derangements. Fundamental theorem of arithmetic, divisibility in Z, congruences, Chinese Remainder Theorem, Euler's Ø- function, primitive roots. Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems. Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain. Polynomial rings and irreducibility criteria. Fields, finite fields, field extensions, Galois Theory.

UNIT - III

Ordinary Differential Equations (ODE):

Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs. General theory of homogenous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green's function.

Partial Differential Equations (PDE):

Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs. Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

UNIT - IV

Numerical Analysis:

Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

Calculus of Variations:

Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema. Variational methods for boundary value problems in ordinary and partial differential equations. Linear Integral Equations: Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels. Characteristic numbers and eigenfunctions, resolvent kernel.

UNIT - V

Topology:

basis, dense sets, subspace and product topology, separation axioms, connectedness and compactness.

Classical Mechanics:

Generalized coordinates, Lagrange's equations, Hamilton's canonical equations, Hamilton's principle and principle of least action, Two-dimensional motion of rigid bodies, Euler's dynamical equations for the motion of a rigid body about an axis, theory of small oscillations. Runge-Kutta methods.